Hollow Ground Geometry

(Revised, March 2009)

Introduction

The many articles on sharpening chisels and plane irons always include some discussion of the bevel angle of the blade. For chisels, it ranges from 20° for paring chisels to 25° for bench chisels to 30° or even 35° for mortise chisels. But how does one accurately set the tool rest to achieve the desired angle? And how does the curvature of the hollow grind come in to play? Years of guessing and trial and error finally prompted the careful examination of the geometry of hollow grinding, with the results used to construct a quick and simple jig to set the grinder tool rest in and accurate and repeatable manner.

Hollow Grind Geometry

The geometry of hollow grinding is illustrated in Figure 1.

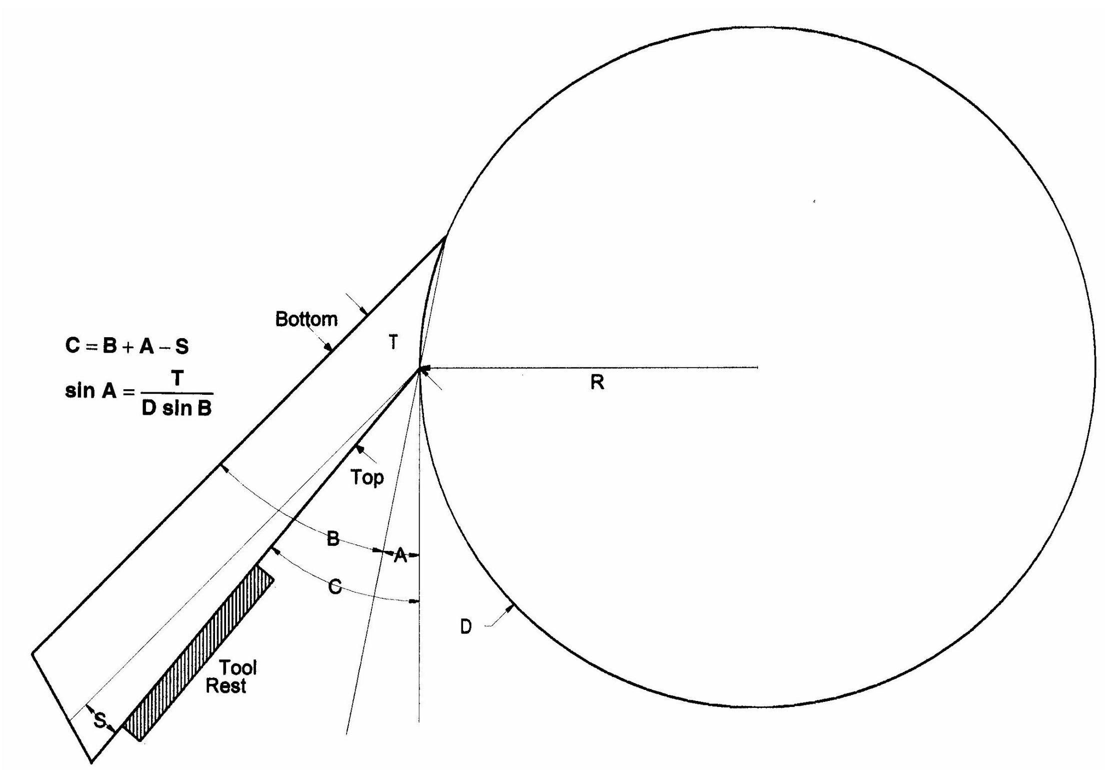


Figure 1 – Geometry of a Hollow Grind

There are many variables involved in this problem which are defined as follows.

B is the desired bevel angle
T is the thickness of the chisel at the end of the blade
R is the radius of the grinding wheel
D is the diameter of the grinding wheel
S is the taper angle of the blade, if any
A is the correction angle to account for the curvature of the hollow grind.
C is the tool rest angle to achieve B

All of these parameters are known for a given grinding problem. The required value for C is then given by

$$C = B + A - S \qquad \text{Eq. (1)}$$

For accuracy, a digital micrometer is suggested for measuring linear parameters. The value for T is then a straightforward measurement.

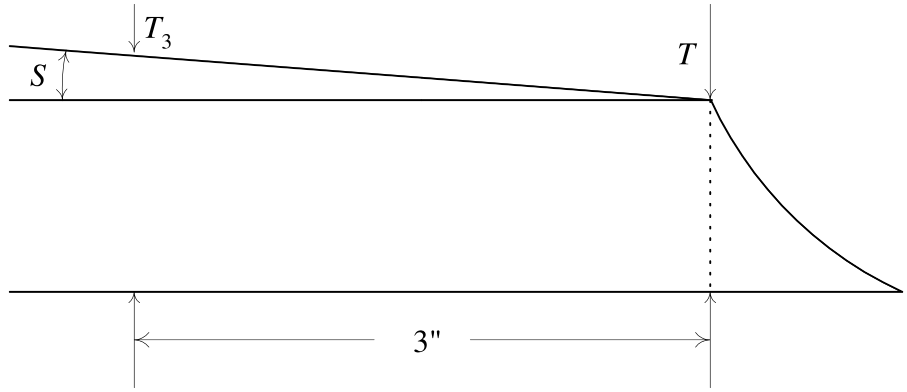


Figure 2 – Measurement of Chisel Taper

The value for *S* is a bit more difficult. Use of a protractor is impractical. A better method is to measure the blade thickness at 3.0" up the blade from the location of the top of the hollow grind where *T* is measured as shown in Figure 2. If we denote this thickness as T_3 then the value for the taper angle *S* will be

$$S = 19\,(T_3 - T) \quad \text{degrees} \qquad \text{Eq. (2)}$$

where *T* is expressed in inches. To obtain this equation we used the approximation that $\tan x \cong x$ for small angles, and one radian is approximately 57º. The measurement of several different chisel sizes and brands showed that a value of 1.5º can be used as a good approximation for the value for *S* for bench chisels.

To find the value for the angle *A* we need some geometry and trigonometry. Figure 3 shows the geometry that relates the angles *A* and *B* and the blade thickness *T*. For clarity the arc of the grindstone which passes through points *a* and *c* is omitted.

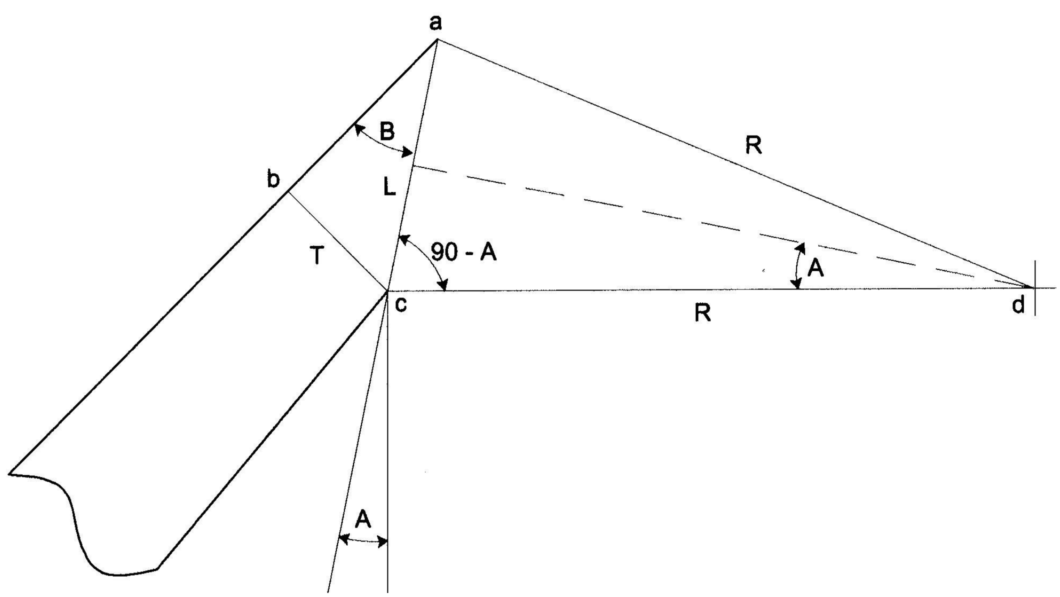


Figure 3 **– Geometry for Determining Angle *A***

The first step is to determine the length *L*, the chord which subtends the hollow grind. This is found by solving the right triangle *abc*. The bevel angle *B* is opposite the side *T*, so the hypotenuse *L* is given by

$$L = \frac{T}{\sin B} \qquad \text{Eq. (3)}$$

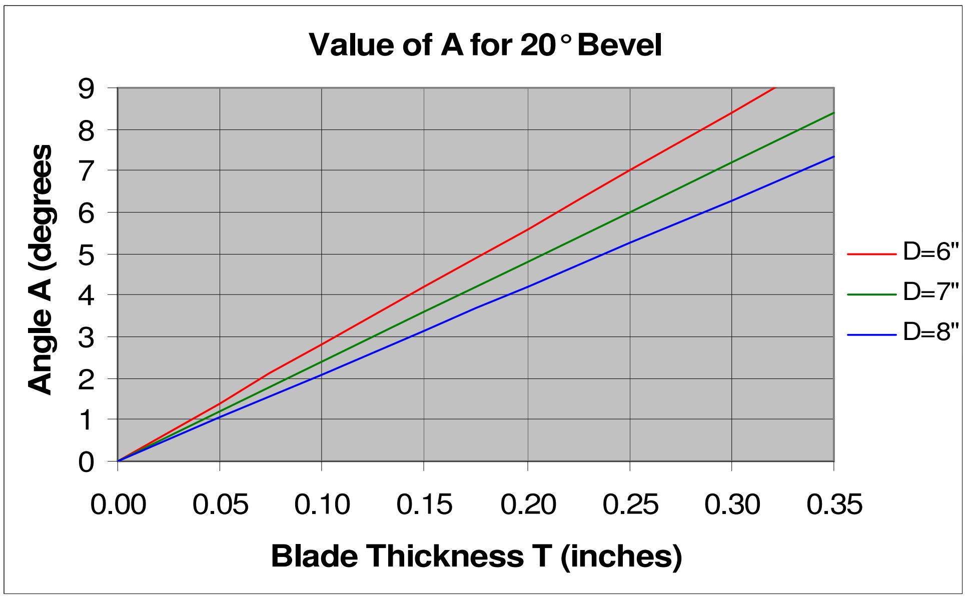
The triangle *adc* is isosceles, with L as it's base and the wheel radius R as it's sides. The angle at each end of the base is $90 - A$, so the total angle at the apex will be $2A$. We drop a dashed line from the apex to the base to bisect the base and produce a right triangle with hypotenuse R. The solution of this triangle is

$$\sin A = \frac{L/2}{R} \qquad \text{Eq. (4)}$$

Eliminating L between Eq. (3) and Eq. (4) yields

$$\sin A = \frac{T}{2R\sin B} \quad \text{or} \quad A = \arcsin\frac{T}{D\sin B} \qquad \text{Eq. (5)}$$

This value for A involves trigonometric functions, so the solution is graphed in Figure 4 for some typical values of D and T and a bevel angles of B = 20º, 25ºand 30º.



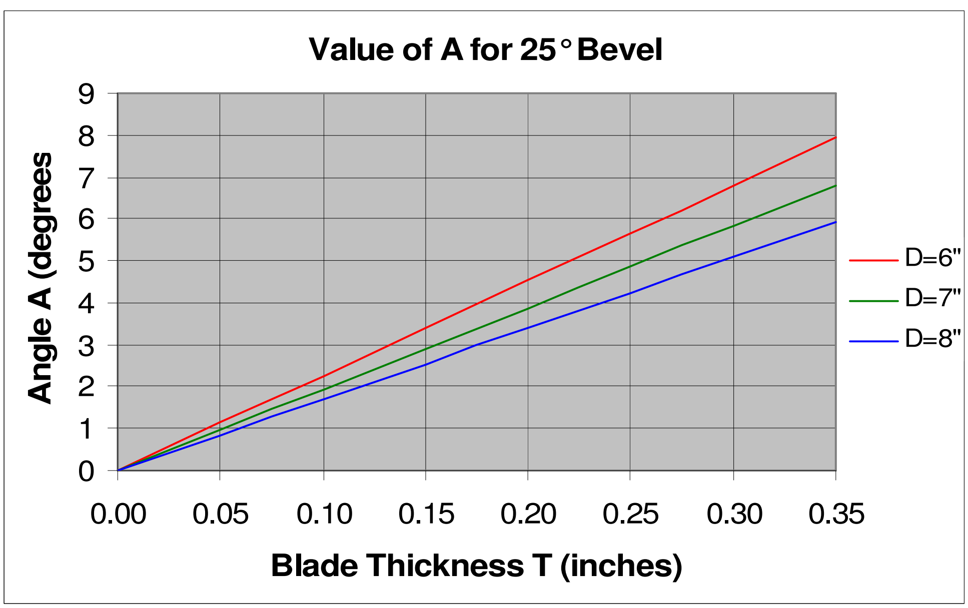


Figure 4a – Correction Angles for 20° and 25° Hollow Grind Curvatures

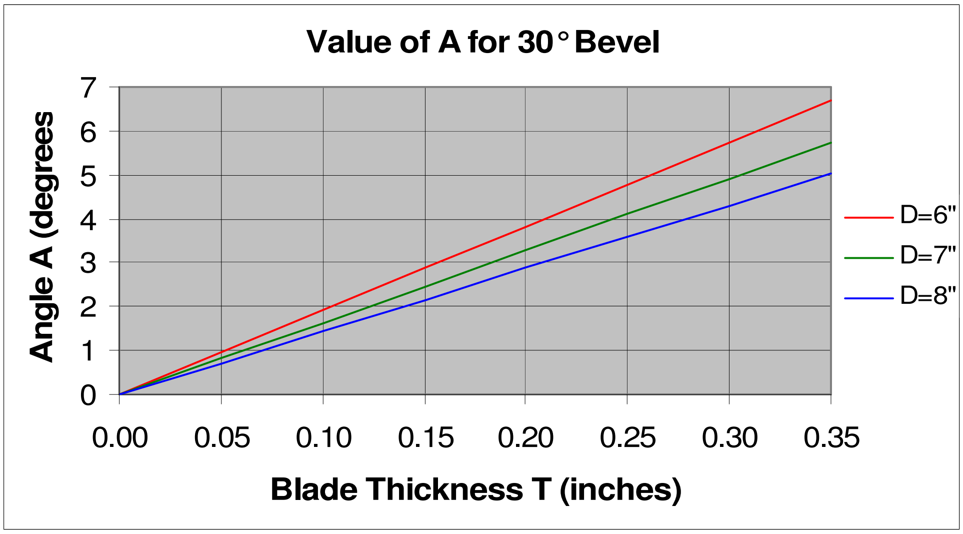


Figure 4b – Correction Angle for 30° Hollow Grind Curvature

Since we will be normally be happy to achieve a bevel angle to the nearest degree, Figure 4 will permit interpolating reasonable correction angles, *A,* for bevel angles between 20º and 30º and blade thickness up to 0.35 inches. If more accuracy is needed for larger bevel angles and blade thickness, then Equation (5) should be solved for the specific value *A* using a scientific calculator.

Since typical values of blade thickness for bench chisels of 1" or less are in the range of 0.15 inches, a value for *A* of 3º is a reasonable approximation for 6" to 8" wheel diameters and a 25º bevel angle. Substitution of the approximate values of $S = 1.5º$ and $A = 3º$ into Eq. (1) shows that one to two degrees should be added to the numerical bevel angle when the tool rest jig is laid out for a bench chisel. For a 25º plane blade the value of *S* is zero and the thickness is about 0.1 inch. Figure 4 then yields a value of *A* of about 2º, which should be added to the desired bevel angle when laying out the jig.

Turning Tools

Turning tools are typically ground to larger bevel angles and are substantially thicker than bench chisels. This increases the correction angle, *A*, which must be added to the desired bevel angle when laying out the tool rest jig.

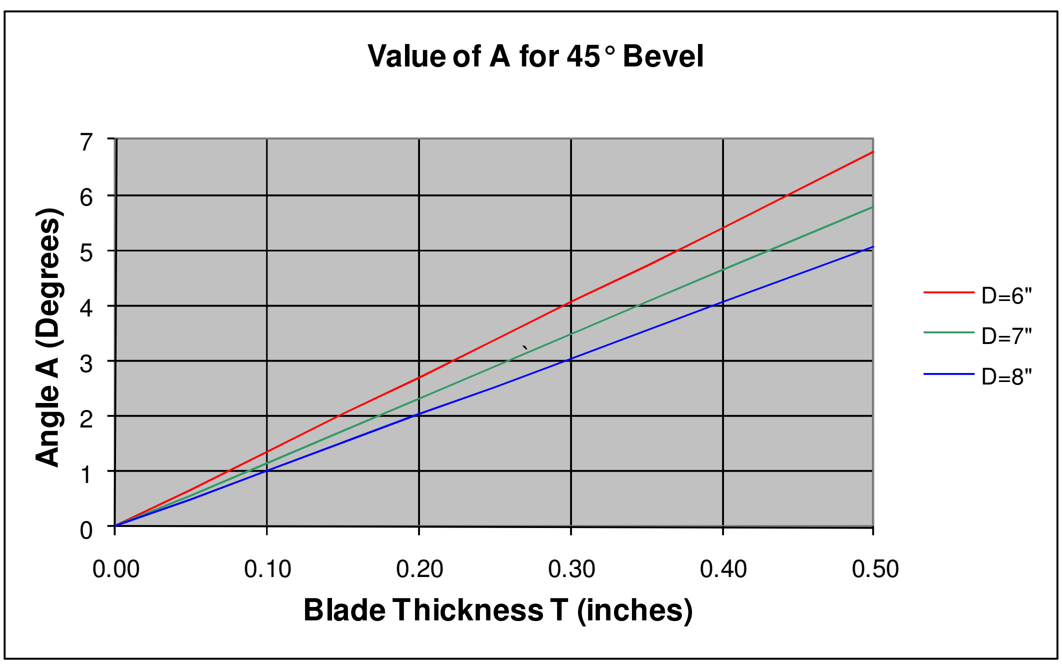


Figure 5 – Correction Angle for 45° Hollow Grind Curvature

Figure 5 shows the correction angle to add to the jig layout for a 45º bevel angle and blade thickness up to 0.5 inch.

A word is in order on determining the blade thickness. For a scraper, the thickness is the complete thickness of the blade. For a skew, the blade thickness is one-half the skew thickness as the edge is symmetrically ground equally on each half of the skew. The hollow ground surface only extends over half the total thickness of the skew.

For gouges, the problem gets more complex, as the angle of the gouge relative to the wheel axis varies as the grind tapers back around the edges. In this case, the proper thickness for the jig calculation would be the thickness of the gouge, and the bevel angle will be correct near the center of the gouge. The sides will have to be done freehand as the sides taper back.

Finally, with a large bevel angle, the midpoint location on the jig layout of the first compass arc in the original article may be too far to the left. Locating the initial compass point one-quarter to one-third the distance from the right edge may be necessary. Note that the essential points are that the flat portion needs only be long enough to cover the tool rest surface, and the arc only long enough to establish a firm position on the grinding wheel. Trim the lengths accordingly to fit your grinder.

For cases where other bevel angles are desired the correction angle for an 8" wheel is shown in Figure 6 as a function of bevel angle for three thicknesses. This should allow an estimate of the value of *A* for any case. Again, if more accuracy is required, then solve Equation (5) with the exact values of *T*, *B*, and *D*.

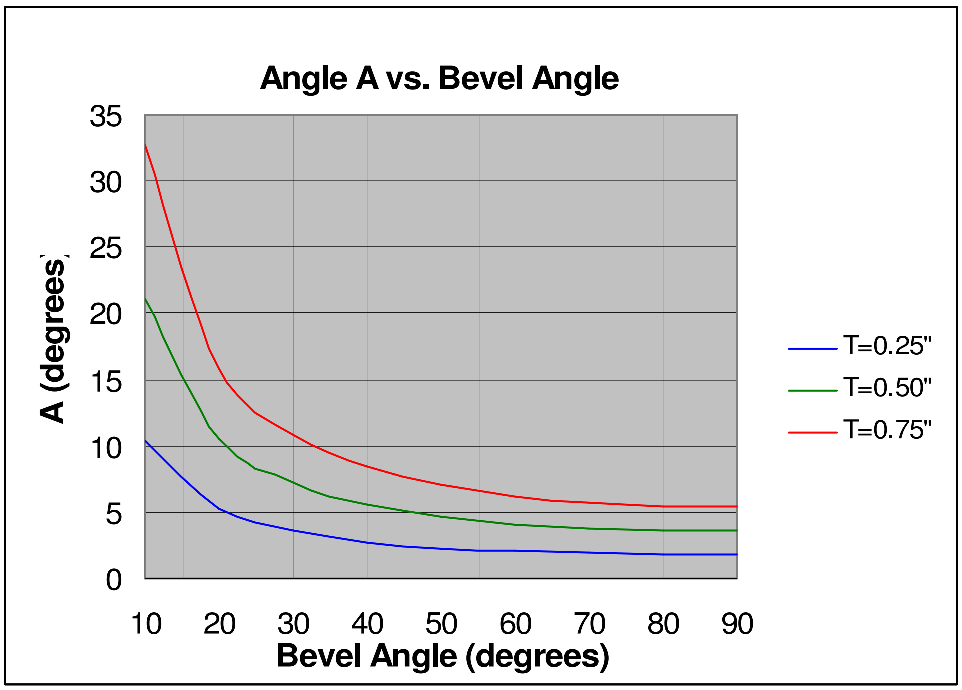


Figure 6 – Correction Angle *vs*. Bevel Angle for 8" Wheel

Conclusions

Analysis of the geometry of hollow ground blades has been presented. For typical bench chisels with a taper, the tool rest should be set one degree greater than desired bevel angle. For plane blades and chisels with no taper the tool rest should be set two degrees greater than the desired bevel angle. For blades that do not meet these approximate thicknesses or tapers, Equations (2) and (5) permit calculation of the exact values of *S* and *A* for the determining the tool rest angle.

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